Mathematical Physics Seminars 2015-2016

All seminars are held in Room M/2.06 on Mondays at 2:10pm unless otherwise stated. All are welcome. Programme Organiser and Contact: Dr Suresh Eswarathasan

28 September 2015

Speaker: Kirill Cherednichenko (Bath)

Title: Boundary triples, Krein formula, and resolvent estimates for one-dimensional high-A1:C1 periodic problems

Abstract: I will discuss operator-norm resolvent convergence estimates for one-dimensional periodic differential operators with rapidly oscillating coefficients in the non-uniformly elliptic high-contrast setting, which has been out of reach of the existing homogenisation techniques. Our analysis is based on a special representation of the resolvent of the operator in terms of the M-matrix of an associated boundary triple, due to M. G. Krein. The resulting asymptotic behaviour is shown to be described, up to a unitary equivalent transformation, by a non-standard version of the Kronig-Penney model on $\mathbb R$. This is joint work with Alexander V. Kiselev.

5 October 2015

Speaker: Alexander V. Sobolev (University College London)

Title: On non-smooth functions of Wiener-Hopf operators

Abstract: We discuss trace formulae for the operator f(PAP) - Pf(A)P, where A is a pseudo-differential operator on $L^2(R^d)$ with a smooth or discontinuous symbol, and P is a multiplication by the indicator of a piece-wise smooth domain in R^d. The function f is not supposed to be smooth. The obtained formulae generalise results obtained by H. Widom in the 80's.These results are used to study the entanglement entropy of free fermions at positive temperature both in the low and high temperature limits.

12 October 2015

Speaker: Colin Guillarmou (ENS - Paris)

Title: On the lens rigidity problem in case of trapping

Abstract: We shall discuss the inverse problem which consists in the determination a Riemannian metric on a manifold with boundary from measurements of the lengths of geodesics joining boundary points.

19 October 2015

Speaker: Yuri Korolev (Queens Mary London)

Title: Regularisation of Linear Ill-posed Problems in Banach Lattices

Abstract: Partial order is a handy tool in deterministic error modelling. Many functional spaces that are common in the practice of solving inverse problems, such as the L^p-spaces, can be equipped with a natural partial order relation, which turns them into Banach lattices - functional spaces with a partial order and a norm that agree with each other in a certain sense.

Partial order can be used in the formulation of an inverse problem to represent errors in the data and the forward operator as bounds by means of the appropriate partial orders. One advantage is that one gets convex fidelity constraints even if the operator is not known exactly - something one does not get with a norm only.

26 October 2015

Speaker: Suresh Eswarathasan (Cardiff)

Title: Quantum chaos and scarring from the viewpoint of microlocal analysis

Abstract: In this talk, we will review some well-known problems in quantum chaos and show how semiclassical microlocal analysis is a useful tool for them. In the remaining portion of the talk, I will present a recent result with S. Nonnenmacher (Université de Paris-Sud) involving generalised eigenfunctions which concentrate around closed geodesics (i.e. scarred states) and how this is related to a generalised version of the Quantum Unique Ergodicity conjecture of Rudnick and Sarnak.

2 November 2015

Speaker: Professor Geoffrey Burton (University of Bath)

Title: Variational problems involving rearrangements of functions

Abstract: Two real functions on a domain in Euclidean space are said to be rearrangements of one another if they have the same decreasing rearrangement, that is, corresponding super-level sets of the two functions always have equal measure. Variational problems where the set of all rearrangements of one fixed function forms the constraint arise in fluid mechanics, where the maximisers of kinetic energy represent steady configurations for a vortex. Existence of maximisers can be proved by means of convex analysis in conjunction with compactness properties of highly symmetric functions. In order to study stability one needs to prove compactness of all maximising sequences, which may lack symmetry. This necessitates the use of concentration-compactness.

9 November 2015

Speaker: Katie Gittins (Bristol)

Title: On the heat content of open sets in R^m.

Abstract: Linked

16 November 2015

Speaker: Valery Smyshlyaev (University College London)

Title: Operators' coercivity in high-frequency scattering

Abstract: Scattering problems are problems on the spectrum, and their solutions are understood as `limited absorption' limits. This implies a great difficulty in controlling inverses of the relevant operators, especially at high frequencies as required for justification of high frequency asymptotics. The latter was intensively advanced starting 1960s using integral equations and non-standard "multiplies" techniques, and later using microlocal analysis. Recent advances in asymptotic-numerical hybrids for high-frequency scattering have posed new challenges for analysis. In particular, for Galerkin methods for boundary integral equations proving error bounds requires stronger results than bounding the inverses, namely bounding the operators' coercivity constants. It appears that an appropriate modification of the multipliers' techniques of Morawetz & Co allows achieving this for certain scatterers. Joint work with Euan Spence (Bath) and Ilia Kamotski (UCL).

23 November 2015

Speaker: Dr Alessio Martini (University of Birmingham)

Title: Sub-Elliptic Operators and Sharp Multiplier Theorem

Abstract: Linked

30 November 2015

Speaker: Elaine Crooks (Swansea)

Title: Compensated convexity, Hausdorff-stable singularity extraction, and image processing

Abstract: Compensated convex transforms enjoy tight-approximation and locality properties that can be exploited to develop multi-scale, parametrised methods for identifying singularities in functions. These tools can then be used, via a numerical

implementation, to detect features in images or data, remove noise from images, identify intersections between surfaces, etc, and thus produce new geometric techniques for image processing, feature extraction and geometric interrogation.

7 December 2015

Speaker: Frederick Symons (Cardiff University)

Title: Uniqueness in an inverse problem for a pencil of operators

Abstract: Linked

25 January 2016

Speaker: Claudia Garreto (Loughborough)

Title: A survey on weakly hyperbolic equations and systems

Abstract: This talk is a survey on some recent work, in collaboration with Michael Ruzhansky (Imperial College London) on weakly hyperbolic equations and systems. The expression weakly refers to the presence of multiple roots/eigenvalues. We will discuss well-posedness for the corresponding Cauchy problem in suitable function spaces and how to deal with low regular coefficients.

1 February 2016

Speaker: Jiang-Lun Wu (Swansea)

Title: Maximum principles for parabolic Waldenfels operators

Abstract: As a sub-class of Levy type Markov generators, second order (elliptic) Waldenfels operators appear naturally when considering the problem of construction of (in particular jump type) Markov processes with boundary conditions. In this talk, parabolic Waldenfels operators will be introduced. Weak and strong maximum principles as well as the boundary point lemma for such operators will be discussed. The talk is based on a joint work with Jinqiao Duan (Illinois Institute of Technology, Chicago/USA) and Qiao Huang (Huazhong University of Science and Technology, Wuhan/China).

8 February 2016

Speaker: Bill Lionheart, (Manchester)

Title: Rich and Non-abelian tomography problems

Abstract: In many tomographic methods a non-scalar measurement is made for each ray giving rise to the possibility of imaging non-scalar fields. In some cases the transport equation along a ray is governed by a variable coefficient system of ODEs and due to coefficient matrices not commuting this cannot be reduced to a simple integral transform, hence the term non-abelian tomography.

Imaging magnetic fields using polarized Neutrons is a new tomographic modality. It is modelled by a non-abelian plane-by-plane ray transform, for which uniqueness of solution is known from the work of Eskin for sufficiently smooth magnetic fields.

In polarized light tomography for a birefringent material measurements are made of the change of the polarization of monochromatic light as it passes through the medium. For weakly birefringent media the propagation of light can be modeled by the Rytov-Sharafutdinov law giving rise to a non-abelian ray transform. For sufficiently small deviations from an isotropic background the linearization of this problem is the Truncated Transverse Ray Transform. We review reconstructions algorithms for this linearized problem in special cases, including range conditions.

We go on to show how strain may be imaged using diffraction tomography of monochromatic x-rays as an application of the transverse ray transform, while Bragg edge tomography with neutrons is governed by the longitudinal ray transform and cannot image interior linear strains.

15 February 2016

Speaker: Lior Silberman, (University of British Columbia)

Title: A Haar component for quantum limits on locally symmetric spaces

Abstract: The "quantum unique ergodicity" problem is the study of the concentration properties of eigenfunctions of the Laplace-Beltrami operator on a compact manifold in the limit of high eigenvalues. It is related to the problem of estimating the eigenfunctions pointwise, and more generally to estimating their restrictions to lower-dimensional subsets.

I will introduce the problem in general, and then describe work with N. Anantharaman (U. Strasbourg) on the case of locally symmetric spaces, where it is believed that the eigenfunctions become uniformly distributed in the root-mean-square sense. We showed that in certain cases the limit measure contains a multiple of the uniform measure as a component.

22 February 2016

Speaker: Aram Karakhanyan, (University of Edinburgh)

Title: Regularity of free boundary in two phase variational problems

Abstract: I will give an overview of the regularity theory of two phase free boundary problems for both variational and viscosity solutions. When the governing PDE is linear, say it is the Laplace operator, then many powerful tools and results are available such as the Alt-Caffarelli-Friedman and Weiss monotonicity formulae. However for nonlinear PDEs the theory is widely open. Recently in a joint work with Serena Dipierro we made some advances in this directions and will discuss our result in the second part of the talk.

24 February 2016

Speaker: David Edmunds, (University of Sussex)

Title: Sobolev spaces and the distance function.

Abstract: The talk will provide a criterion, involving the distance function, for detecting functions in Sobolev spaces that have zero trace.

29 February 2016

Speaker: Roman Schubert, (University of Bristol)

Title: What is the semiclassical limit of non-Hermitian time evolution?

Abstract: It is well known that in the semiclassical limit (or the high frequency limit) solutions to many PDE's in physics and other sciences are driven by a Hamiltonian flow, i.e., by solutions to a system of ODE's. Examples include Maxwell's equations and the Schroedinger equation. In this setting the propagation of waves can be described geometrically as the propagation of Lagrangian submanifolds of a symplectic manifold by the Hamiltonian flow.

We will consider the case that the Hamiltonian flow is generated by a complex valued Hamiltonian function, which corresponds to a system with loss or gain and a Schroedinger equation where the Hamilton operator is non-Hermitian. We derive a new class of ODE's which govern the semiclassical limit, and which combine a Hamiltonian vector field, generated by the real part of the Hamiltonian function, with a gradient vector field, generated by the imaginary part. It turns out that the propagation can again be described in terms of symplectic geometry, but this time it includes a complex structure which is generated by the dynamics along the rays. We will give an overview of these new geometric structures emerging from the semiclassical limit. This is joint work with Eva-Maria Graefe.

4 March 2016

Speaker: Micah Milliinovich (University of Mississippi)

Title: Fourier analysis and the zeros of the Riemann zeta-function

Abstract: I will show how the classical Beurling-Selberg extremal problem in harmonic analysis arises naturally when studying the vertical distribution of the zeros of the Riemann zeta-function and other L-functions. Using this relationship, along with techniques from Fourier analysis and reproducing kernel Hilbert spaces, we can prove the sharpest known bounds for the number of zeros in an interval on the critical line and we can also study the pair correlation of zeros. Our results on pair correlation extend earlier work of P. X. Gallagher and give some evidence for the well-known conjecture of H. L. Montgomery. This talk is based on a series of papers which are joint with E. Carneiro, V. Chandee, and F. Littmann.

7 March 2016

Speaker: Jonathan Ben-Artzi (Imperial College London)

Title: Spectral methods in ergodic theory: obtaining uniform ergodic theorems

Abstract: Von Neumann's original proof of the ergodic theorem for one-parameter families of unitary operators relies on a delicate analysis of the spectral measure of the associated flow operator and the observation that over long times only functions that are invariant under the flow make a contribution to the ergodic integral. In this talk I shall show that for a specific class of generators -- namely vector fields -- the spectral measure is rather simple to understand via a Fourier transform. This allows us to obtain a uniform ergodic theorem (on an appropriate subspace). The analysis is performed in both Sobolev and weighted-Sobolev spaces. These results are closely related to recent results on the 2D Euler equations, and have applications to other conservative flows, such as those governed by the Vlasov equation (modeling plasmas and galactic dynamics, for instance).

9 March 2016

Speaker: Matthias Langer (Strath)

Title: Sturm-Liouville operators with two singular endpoints

Abstract: The classical theory of Sturm-Liouville equations deals often with equations that have at most one singular endpoint (roughly speaking, a regular endpoint is one where the initial value problem is well defined). A central object in the theory is the Titchmarsh-Weyl coefficient that is defined with the help of two solutions that satisfy some initial conditions at a regular endpoint.

In the last decade there has been a lot of interest in equations with two singular endpoints. In certain situations, when the coefficients at one of the endpoints are not too singular, one

can use some regularisation procedure to fix two solutions and define a singular Titchmarsh-Weyl coefficient. With the latter one can construct a spectral measure, show that the spectral multiplicity is one and prove direct and inverse spectral theorems.

14 March 2016

Speaker: Andrew Granville (UCL)

Title: The pretentious Riemann Hypothesis and beyond

Abstract: We give some insights into the "alternative approach" to analytic number theory being developed by Soundararajan and the speaker, for example using simple ideas from a first complex analysis course to state version of the Riemann Hypothesis that does not involve zeros of the Riemann zeta function, nor its analytic continuation.

16 March 2016

Speaker: Martin Dindos (Edinburgh)

Title: Boundary value problems for elliptic PDEs with coefficients satisfying Carleson condition

Abstract: I will talk on two papers, the first one is an older work with J. Pipher and S. Petermichl and the second one is a joint work with J. Pipher and D. Rule. I will introduce the L^p Dirichlet, Regularity and Neumann problems for elliptic PDEs with bounded measurable coefficients. In general without any additional condition there are examples of elliptic operators such that these BVP are not solvable for any 1 assumption on the coefficients and proceed to present results on solvability of these BVP under this extra conditions.

At the end I will mention few open problems and briefly talk about possible generalizations of these results to elliptic systems.

11 April 2016

Speaker: Anthony Ashton (DAMTP Cambridge) CANCELLED

Title: Unified approach to elliptic boundary value problems: Overview and recent developments

Abstract: In the 1990s Thanasis Fokas introduced a new approach for boundary value problems associated with nonlinear, integrable PDEs. Remarkably, ideas used in this

approach could also be utilised in the study of linear, elliptic PDEs. I will give an overview of this unified method, in the context of elliptic boundary value problems, and discuss recent developments.

18 April 2016

Speaker: Sergey Naboko (St. Petersburg)

Title: On the structure of operator-valued Herglotz functions and applications

Abstract: We will discuss some properties of the operator-valued Herglotz functions appearing in the perturbation theory of self-adjoint and non-self-adjoint operators on Hilbert space. Applications will be discussed.

25 April 2016

Speaker: Robert Leek (Cardiff University)

Title: A structural theory of convergence

Abstract: Sequences occur prominently throughout analysis, even in spaces which are not metrisable, nor even first-countable. It would thus appear useful to understand when a topology can be `described' via its convergent sequences.

In this talk, I will discuss Fréchet-Urysohn spaces (spaces where the closure of a set is precisely the limits of sequences within that set) and spokes and spoke systems, which provide a local structure that characterises this property. I will discuss some basic properties of spokes, with plenty of examples, and show that (with a weak separation axiom) we can enhance the structure by using closed spokes. This gives a nice internal language to discuss Fréchet-Urysohn compactifications, as well as algebraic characterisations for ring spectra (e.g. Stone spaces).

No knowledge beyond basic definitions in general topology will be assumed.

4 May 2016

Speaker: Xue-Mei Li (Warwick)

Title: Perturbation to Conservation Laws

Abstract: I discuss stochastic averaging and homogenization problems arising from mathematical physics and geometry. These will be viewed as perturbation to conservation laws, which are not necessarily real valued functions, nor necessarily taking values in a vector space.

Examples include perturbation to Hamitlonian systems (including the geodesic equation) and collapsing of Riemannian manifolds by scaling Riemanian metrics, and also families of second order differential operators in the form of the sum of non-commuting operators.